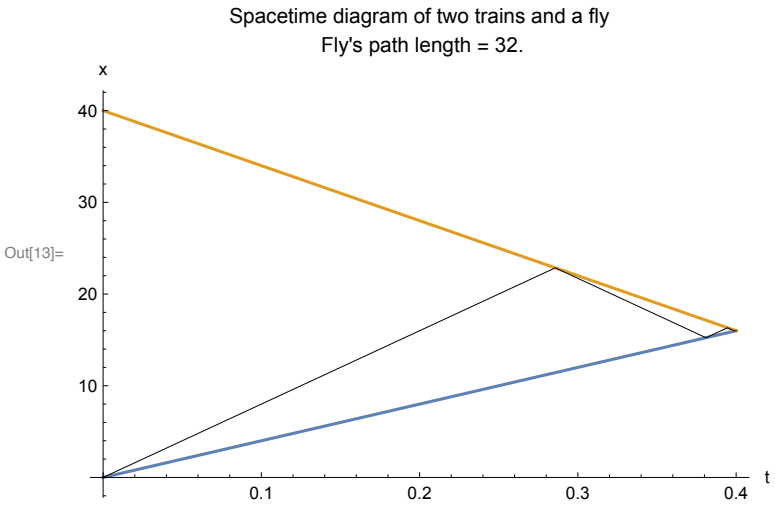
In[1]:=
```
flyTurnCoords[f_, g_, Vfly_, nTurns_] :=
 (* f[t] is the position of car 1 at time t. f[0] is assumed to be 0.
   g[t] is the position of car 2 at time t.
    Vfly is a order pair {Vright, Vleft} =
  fly's velocities to the right and to the left.
    nTurns is the number of time to calculate fly's trajectory.
    The fly's initial position is assumed to be at (0, f[0]).
    This function returns {t, x} coordinates that the fly meets the cars.
 *)
 Module[{V1, V2, t0, x0 , pt , coords = {}},
  {V1, V2} = Vfly;
  {t0, x0} = {0, f[0]};
  pt = {t0, x0};
  coords = Append[coords, pt];
  For[turn = 1, turn ≤ nTurns, turn ++,
   pt = {t, g[t]} /. FindRoot[x0 + V1 (t - t0) == g[t], {t, t0}];
   coords = Append[coords, pt];
   {t0, x0} = pt;
   pt = {t, f[t]} /. FindRoot[x0 + V2 (t - t0) == f[t], {t, t0}];
   coords = Append[coords, pt];
   {t0, x0} = pt;];
  coords
 ]

pathLength[coords_] := Apply[Plus, Abs[Differences[Transpose[coords][[2]]]]];
(* pathLength[coords_] calculates the distance that the
 fly flies between points specified in coords.  coords should
 be the {t,x} coordinates obtained from flyTurnCoords[ ] *)

flyPathLength[T_, d_, V1_, V2_] := (d (V1 - V2) + 2 V1 V2 T) / (V1 + V2)
(*Distance that the fly travels between the two cars untill the
 collision.  The collision is assumed to be at distance d from the
 origin. The time of the collision is assumed to be at T. V1 is
 the fly's speed to the right. V2 is the fly' speed to the left.*)
flyPathLength[T_, d_, Vfly_] := Module[{V1, V2},
  (* If V1, V2 is implicitly passed in Vfly vector,
  use this version of function. *)
  {V1, V2} = Abs[Vfly];
  flyPathLength[T, d, V1, V2]]

showFlyFlight[f_, g_ , Vfly_, tmax_, iterations_] :=
 (* Draw the positions of car 1, car 2, and the fly. *)
 Module[{flightLength, coords,  g1, g2},
  coords = flyTurnCoords[f, g , Vfly, iterations];
  g1 = Plot[{f[t], g[t]}, {t, 0, tmax}, PlotLabel →
      "Spacetime diagram of two trains and a fly\n Fly's path length = " <>
```

```
      ToString[pathLength[coords]], AxesLabel → {"t", "x"}];
  g2 = Graphics[Line[coords]];
  (*Print["Path length = ",pathLength[coords]];*)
  Show[g1, g2]]
```

In[6]:=
```
(* Testing with car 1 moving to the right at 40 kmph,
car 2 moving to the left at 60 kmph. Car
 2's initial position is 40 km away from car 1.
 The fly flies at 80 kmph to both right and left.*)
f[t_] := 0 + 40 t;
g[t_] := -60 t + 40;
Vfly = {80, -80};
coords = flyTurnCoords[f, g , Vfly, 5];
(* Find 5 rounds of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[40 / (40 + 60), 40 × 40 / (40 + 60), 80, 80],
  " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 40 / (40 + 60), 5]
```

```
The fly flies a total of 32 km. (Given by theoretical formula.)
The {t,x} coordinates that the fly meets the car:
 {{0, 0}, {0.285714, 22.8571}, {0.380952, 15.2381}, {0.394558, 16.3265},
  {0.399093, 15.9637}, {0.399741, 16.0155}, {0.399957, 15.9983},
  {0.399988, 16.0007}, {0.399998, 15.9999}, {0.399999, 16.}, {0.4, 16.}}
Explicitly sum the flight distance from the coordinates above: 32. km.
```

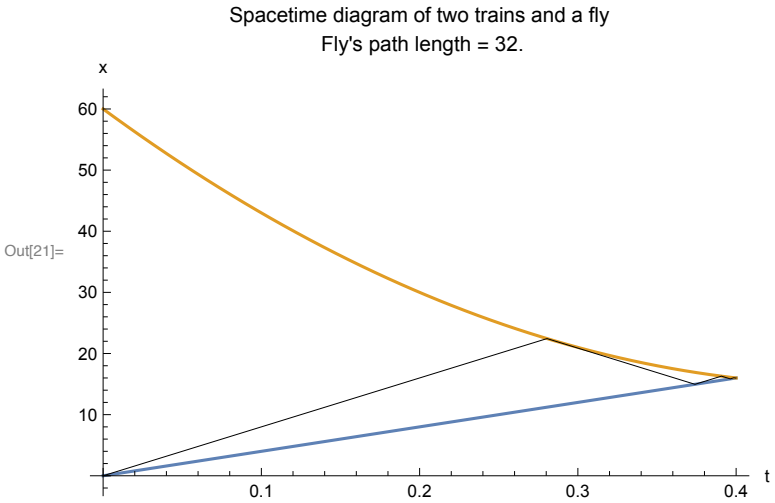
Out[13]=



In[14]:=
```
f[t_] := 0 + 40 t;
g[t_] := InterpolatingPolynomial[{{0, 60}, {0.2, 30}, {0.4, 16}}, t]
Vfly = {80, -80};
coords = flyTurnCoords[f, g , Vfly, 8];
(* Find 8 rounds of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[40 / (40 + 60), 40 × 40 / (40 + 60), Vfly],
  " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 40 / (40 + 60), 8]
```

```
The fly flies a total of 32 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0}, {0.280507, 22.4405}, {0.374009, 14.9604}, {0.390706, 16.2961},
  {0.396271, 15.8509}, {0.398647, 16.0409}, {0.399439, 15.9776}, {0.399796, 16.0061},
  {0.399915, 15.9966}, {0.399969, 16.0009}, {0.399987, 15.9995}, {0.399995, 16.0001},
  {0.399998, 15.9999}, {0.399999, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}}

Explicitly sum the flight distance from the coordinates above: 32. km.
```

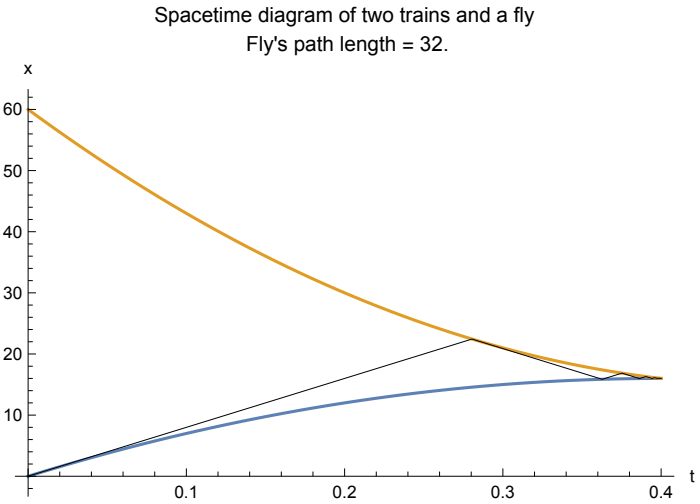
Out[21]=



In[22]:=
```
f[t_] := InterpolatingPolynomial[{{0, 0}, {0.2, 12}, {0.4, 16}}, t];
g[t_] := InterpolatingPolynomial[{{0, 60}, {0.2, 30}, {0.4, 16}}, t]
Vfly = {80, -80};
coords = flyTurnCoords[f, g , Vfly, 20];
(* Find 20 rounds of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[40 / (40 + 60), 40 × 40 / (40 + 60), Vfly],
  " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 40 / (40 + 60), 20]
```

```
The fly flies a total of 32 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0.}, {0.280507, 22.4405}, {0.362748, 15.8612}, {0.37528, 16.8638},
  {0.386312, 15.9813}, {0.390384, 16.307}, {0.394262, 15.9967}, {0.395888, 16.1268},
  {0.39748, 15.9994}, {0.398179, 16.0553}, {0.398872, 15.9999}, {0.399182, 16.0247},
  {0.399491, 16.}, {0.39963, 16.0111}, {0.399769, 16.}, {0.399832, 16.005},
  {0.399895, 16.}, {0.399924, 16.0023}, {0.399952, 16.}, {0.399965, 16.001},
  {0.399978, 16.}, {0.399984, 16.0005}, {0.39999, 16.}, {0.399993, 16.0002},
  {0.399996, 16.}, {0.399997, 16.0001}, {0.399998, 16.}, {0.399999, 16.},
  {0.399999, 16.}, {0.399999, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.},
  {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}}

Explicitly sum the flight distance from the coordinates above: 32. km.
```

Out[29]=

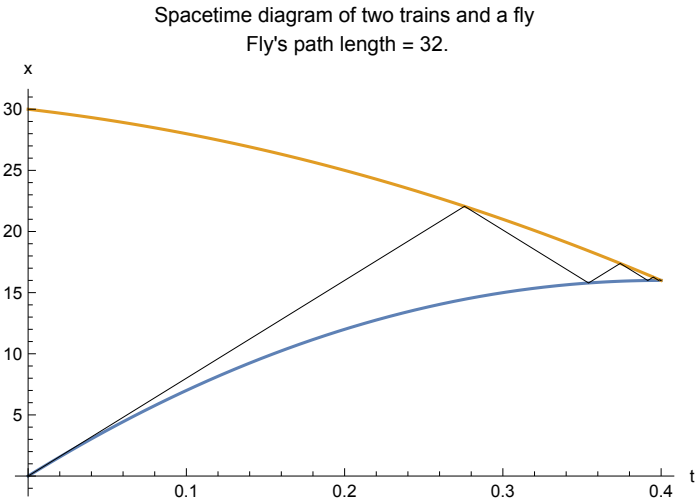


In[30]:=

```
f[t_] := InterpolatingPolynomial[{{0, 0}, {0.2, 12}, {0.4, 16}}, t];
g[t_] := InterpolatingPolynomial[{{0, 30}, {0.2, 25}, {0.4, 16}}, t]
Vfly = {80, -80};
coords = flyTurnCoords[f, g , Vfly, 20];
(* Find 20 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[40 / (40 + 60), 40 × 40 / (40 + 60), Vfly],
  " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 40 / (40 + 60), 20]
```

```
The fly flies a total of 32 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0.}, {0.275765, 22.0612}, {0.354157, 15.7898}, {0.374143, 17.3887},
  {0.39159, 15.9929}, {0.39506, 16.2705}, {0.398444, 15.9998}, {0.399079, 16.0506},
  {0.399712, 16.}, {0.399829, 16.0094}, {0.399947, 16.}, {0.399968, 16.0017},
  {0.39999, 16.}, {0.399994, 16.0003}, {0.399998, 16.}, {0.399999, 16.0001},
  {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.},
  {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.},
  {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.},
  {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}, {0.4, 16.}}

Explicitly sum the flight distance from the coordinates above: 32. km.
```

Out[37]=

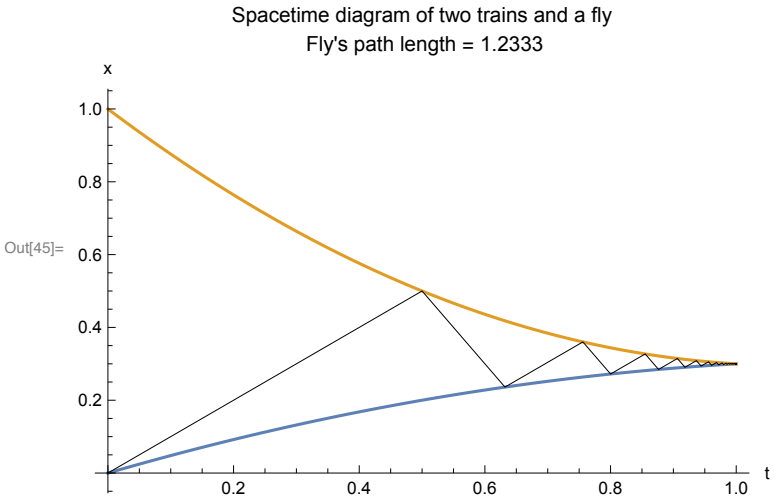


In[38]:=

```
f[t_] := InterpolatingPolynomial[{{0, 0}, {0.5, 0.2}, {1, 0.3}}, t];
g[t_] := InterpolatingPolynomial[{{0, 1}, {0.5, 0.5}, {1, 0.3}}, t]
Vfly = {1, -2};
coords = flyTurnCoords[f, g , Vfly, 30];
(* Find 30 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[1, 0.3, Vfly], " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 1, 30]
```

```
The fly flies a total of 1.23333 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0.}, {0.5, 0.5}, {0.631949, 0.236103}, {0.755977, 0.360131}, {0.800039, 0.272007},
  {0.855115, 0.327083}, {0.876367, 0.28458}, {0.906403, 0.314616}, {0.918454, 0.290515},
  {0.936676, 0.308738}, {0.94415, 0.293791}, {0.955931, 0.305572}, {0.960829, 0.295776},
  {0.968762, 0.303709}, {0.97209, 0.297053}, {0.977581, 0.302544}, {0.979898, 0.297909},
  {0.98377, 0.301781}, {0.985411, 0.298499}, {0.988179, 0.301266}, {0.989355, 0.298913},
  {0.991352, 0.30091}, {0.992203, 0.299208}, {0.993653, 0.300659}, {0.994273, 0.299421},
  {0.995332, 0.30048}, {0.995784, 0.299575}, {0.99656, 0.300351}, {0.996892, 0.299687},
  {0.997463, 0.300258}, {0.997707, 0.29977}, {0.998126, 0.300189}, {0.998306, 0.29983},
  {0.998616, 0.30014}, {0.998748, 0.299875}, {0.998977, 0.300103}, {0.999075, 0.299907},
  {0.999243, 0.300076}, {0.999316, 0.299931}, {0.99944, 0.300056},
  {0.999494, 0.299949}, {0.999586, 0.300042}, {0.999625, 0.299963},
  {0.999694, 0.300031}, {0.999723, 0.299972}, {0.999773, 0.300023},
  {0.999795, 0.299979}, {0.999832, 0.300017}, {0.999848, 0.299985},
  {0.999876, 0.300012}, {0.999888, 0.299989}, {0.999908, 0.300009},
  {0.999917, 0.299992}, {0.999932, 0.300007}, {0.999938, 0.299994},
  {0.99995, 0.300005}, {0.999954, 0.299995}, {0.999963, 0.300004},
  {0.999966, 0.299997}, {0.999972, 0.300003}, {0.999975, 0.299998}}

Explicitly sum the flight distance from the coordinates above: 1.2333 km.
```

Out[45]=

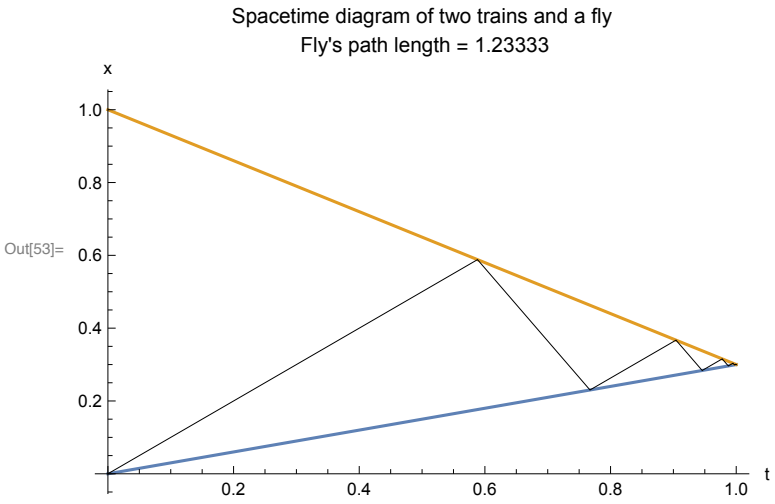


In[46]:=

```
f[t_] := InterpolatingPolynomial[{{0, 0}, {0.5, 0.15}, {1, 0.3}}, t];
g[t_] := InterpolatingPolynomial[{{0, 1}, {0.5, 0.65}, {1, 0.3}}, t]
Vfly = {1, -2};
coords = flyTurnCoords[f, g , Vfly, 30];
(* Find 30 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[1, 0.3, Vfly], " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 1, 30]
```

```
The fly flies a total of 1.23333 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0.}, {0.588235, 0.588235}, {0.767263, 0.230179}, {0.904167, 0.367083},
  {0.945834, 0.28375}, {0.977696, 0.315613}, {0.987394, 0.296218}, {0.994809, 0.303634},
  {0.997066, 0.29912}, {0.998792, 0.300846}, {0.999317, 0.299795}, {0.999719, 0.300197},
  {0.999841, 0.299952}, {0.999935, 0.300046}, {0.999963, 0.299989},
  {0.999985, 0.300011}, {0.999991, 0.299997}, {0.999996, 0.300002},
  {0.999998, 0.299999}, {0.999999, 0.300001}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}}

Explicitly sum the flight distance from the coordinates above: 1.23333 km.
```

Out[53]=

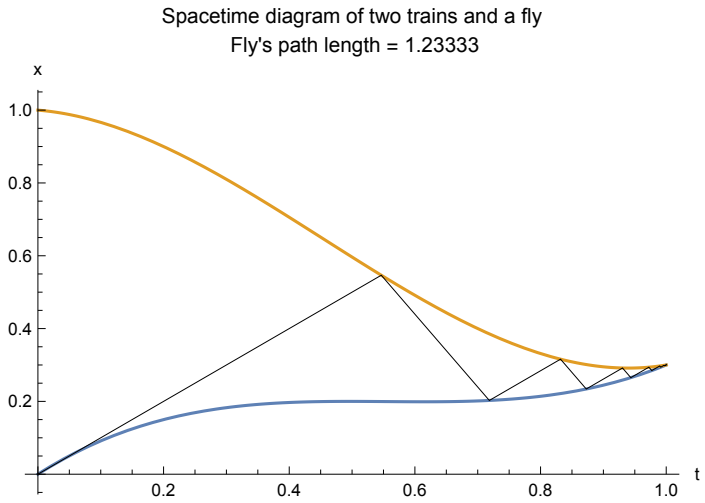


In[54]:=

```
f[t_] :=
  InterpolatingPolynomial[{{0, 0}, {0.2, 0.15}, {0.5, 0.20}, {1, 0.3}}, t];
g[t_] := InterpolatingPolynomial[{{0, 1}, {0.2, 0.9}, {0.7, 0.4}, {1, 0.3}}, t]
Vfly = {1, -2};
coords = flyTurnCoords[f, g , Vfly, 30];
(* Find 30 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[1, 0.3, Vfly], " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 1, 30]
```

```
The fly flies a total of 1.23333 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:
 {{0, 0.}, {0.546503, 0.546503}, {0.718456, 0.202598}, {0.831782, 0.315924},
  {0.872739, 0.234012}, {0.930436, 0.291709}, {0.943672, 0.265236},
  {0.972381, 0.293945}, {0.977024, 0.284658}, {0.989656, 0.29729}, {0.991302, 0.293997},
  {0.996265, 0.298959}, {0.996847, 0.297796}, {0.998674, 0.299622},
  {0.998879, 0.299212}, {0.999532, 0.299866}, {0.999604, 0.299721},
  {0.999835, 0.299953}, {0.999861, 0.299902}, {0.999942, 0.299983},
  {0.999951, 0.299965}, {0.99998, 0.299994}, {0.999983, 0.299988}, {0.999993, 0.299998},
  {0.999994, 0.299996}, {0.999997, 0.299999}, {0.999998, 0.299998},
  {0.999999, 0.3}, {0.999999, 0.299999}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3},
  {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}, {1., 0.3}}

Explicitly sum the flight distance from the coordinates above: 1.23333 km.
```

Out[61]=



In[62]:=

```
f[t_] := Log[t + 1] / (Log[2] / 0.3);
g[t_] := 1 + 1 / 4 Log[t + 1] - 1.8176320469261664` Log[t + 1] ^ 2;
Vfly = {1, -2};
coords = flyTurnCoords[f, g , Vfly, 30];
(* Find 30 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[1, 0.3, Vfly], " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 1, 30]
```

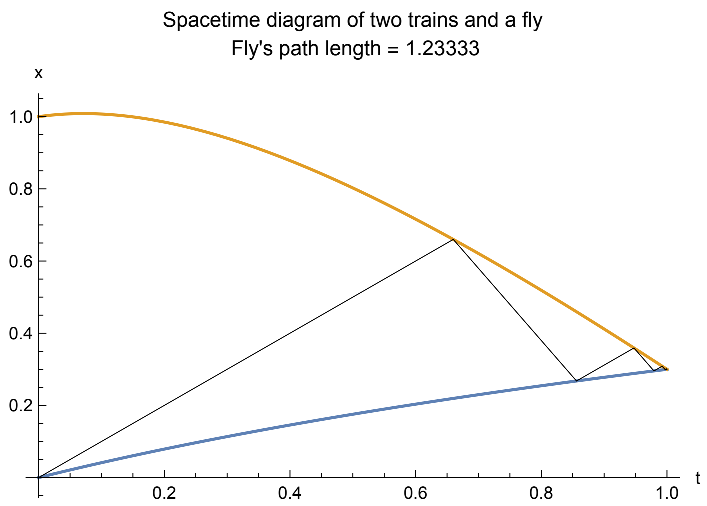
The fly flies a total of 1.23333 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:

{{0, 0.}, {0.659908, 0.659908}, {0.856029, 0.267666}, {0.947479, 0.359115}, {0.979289, 0.295495}, {0.992404, 0.30861}, {0.997031, 0.299357}, {0.99891, 0.301236}, {0.999575, 0.299908}, {0.999844, 0.300177}, {0.999939, 0.299987}, {0.999978, 0.300025}, {0.999991, 0.299998}, {0.999997, 0.300004}, {0.999999, 0.3}, {1., 0.300001}, {1., 0.3}}

Explicitly sum the flight distance from the coordinates above: 1.23333 km.

Out[69]=



In[70]:=

```
f[t_] := Log[t + 1] / (Log[2] / 0.3);
g[t_] := 1 + 1 / 4 Log[t + 1] - 1.8176320469261664` Log[t + 1] ^ 2;
Vfly = {4, -8};
coords = flyTurnCoords[f, g , Vfly, 30];
(* Find 30 round of the fly meeting both cars *)
Print["The fly flies a total of ",
  flyPathLength[1, 0.3, Vfly], " km. (Given by theoretical formula.)"];
Print["The {t,x} coordinates that the fly meets the car: ", coords];
Print["Explicitly sum the flight distance from the coordinates above: ",
 pathLength[coords], " km."]
showFlyFlight[f, g, Vfly, 1, 30]
```

The fly flies a total of 5.23333 km. (Given by theoretical formula.)

The {t,x} coordinates that the fly meets the car:

```
{{0, 0.}, {0.242182, 0.96873}, {0.347152, 0.128974}, {0.512892, 0.791936},
 {0.586901, 0.199864}, {0.693705, 0.627077}, {0.74206, 0.240238}, {0.809299, 0.509197},
 {0.839961, 0.263903}, {0.881852, 0.431469}, {0.901032, 0.278035},
 {0.926995, 0.381889}, {0.93891, 0.286574}, {0.954956, 0.350761}, {0.96233, 0.29177},
 {0.972232, 0.331379}, {0.976786, 0.294947}, {0.982891, 0.319366},
 {0.9857, 0.296894}, {0.989462, 0.311941}, {0.991193, 0.29809}, {0.99351, 0.307358},
 {0.994577, 0.298825}, {0.996004, 0.304533}, {0.996661, 0.299277},
 {0.997539, 0.302792}, {0.997944, 0.299555}, {0.998485, 0.301719},
 {0.998734, 0.299726}, {0.999067, 0.301058}, {0.999221, 0.299831},
 {0.999426, 0.300652}, {0.99952, 0.299896}, {0.999646, 0.300401}, {0.999705, 0.299936},
 {0.999782, 0.300247}, {0.999818, 0.299961}, {0.999866, 0.300152},
 {0.999888, 0.299976}, {0.999917, 0.300094}, {0.999931, 0.299985},
 {0.999949, 0.300058}, {0.999958, 0.299991}, {0.999969, 0.300035},
 {0.999974, 0.299994}, {0.999981, 0.300022}, {0.999984, 0.299997},
 {0.999988, 0.300013}, {0.99999, 0.299998}, {0.999993, 0.300008},
 {0.999994, 0.299999}, {0.999996, 0.300005}, {0.999996, 0.299999},
 {0.999997, 0.300003}, {0.999998, 0.299999}, {0.999998, 0.300002}, {0.999999, 0.3},
 {0.999999, 0.300001}, {0.999999, 0.3}, {0.999999, 0.300001}, {0.999999, 0.3}}
```

Explicitly sum the flight distance from the coordinates above: 5.23333 km.

Out[77]=